Computer Algebra Systems & Artificial Intelligence

Kostas ZOTOS [1]

[1] South-west University "Neofit Rilski", Faculty of Science and Mathematics, Department of Informatics, 66 Ivan Michailov st. 2700 Blagoevgrad, Bulgaria, zwtos@yahoo.com

Abstract: *From four-function calculators to calculators (or computers) with Computer Algebra System (CAS) software, Mathematics computing technology has advanced. With just a few button pushes, CASs can solve a wide range of mathematical problems, which is a true quantum leap in technology. The implications of having software in the classroom that can, for example, expand and factorize algebraic expressions, solve equations, differentiate functions, and find anti-derivatives are causing the mathematical community to engage in a heated debate about whether this is one of the most exciting or frightening developments in the history of education.*

It was only a matter of time before Artificial Intelligence entered the field of Science. This is now also the case with Mathematics, one of the dominant, perhaps the most basic, but also the most "difficult" of the sciences. The human mind, for better or for worse, has its limits. As we see in every manifestation of our lives, in this case, technology is being enlisted to help humanity take the next step, whether it has to do with automation and practical matters, or with knowledge and exploration. Creating a model that is understandable to humans is the primary objective of Artificial Intelligence. Additionally, concepts and methods from numerous mathematical fields can be used to prepare these models. In this paper, we will examine the use of AI in CASs and explore some ways to optimize them. The documentation sheets are the data source that we used to examine their characteristics. The research results reveal that there are many tips that we can follow to accelerate performance.

Keywords: *Computer Algebra Systems; CAS; CAS performance; AI; MATLAB; Maple; Mathematica.*

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Introduction

Numerous academic fields, including Biology, Physics, Economics, Chemistry, and many more, use Mathematics. Many scientific domains have been profoundly impacted by computer technology, which has also improved people's daily lives. Given this, the development of technology could continue to have no impact on the field of Mathematics. A wide range of mathematical software has been developed in recent decades to enhance the comprehension and familiarity of interested users.

From four-function calculators to calculators (or computers) with Computer Algebra System (CAS) software, Mathematics computing technology has advanced. With just a few button pushes, CASs can solve a wide range of mathematical problems, which is a true quantum leap in technology. The implications of having software in the classroom that can, for example, expand and factorize algebraic expressions, solve equations, differentiate functions, and find anti-derivatives are causing the mathematical community to engage in a heated debate about whether this is one of the most exciting or frightening developments in the history of education.

Any mathematical software that can solve algebraic expressions and carry out symbolic mathematical operations is called a **Computer Algebra System (CAS) or Symbolic Algebra System (SAS)**. Since their introduction in the early 1970s, computer algebra systems have undergone significant development.

AI and mathematics

Artificial Intelligence (AI) enables CASs to efficiently understand Mathematics. Recent research has shown that machine learning methods such as support vector machines can improve the performance of CASs through the use of sample problems. Furthermore, inventive CAS applications may result from fresh insights into symbolic computation brought about by explainable AI techniques (Agudelo-Calle et al., 2012). It has also been suggested to employ artificial cognitive systems (ACS) in Mathematics education, with the idea that ACS can act as instruments for cognitive restructuring. These ACS tools, (like, for example, Mathcad), can help with understanding knowledge generated by using them in math education (Pickering et al., 2024).

For the first time in 2021, mathematicians have been able to work with artificial intelligence to propose and prove new mathematical theorems.

The work was done as part of a collaboration between the University of Oxford, the University of Sydney in Australia, and DeepMind (Google's AI company). The developed algorithms can sort data up to three times faster than human-generated versions. The results of this work suggest that machine learning can complement mathematical research (Davies et al., 2021).

Artificial intelligence attempts not only to understand but also to systematize and automate mental tasks. AI emerged from the combination of machine learning and big data, enabling machines to perform computational and other functions like humans. Machine learning is the ability to use experience to improve cognitive behavior, using artificial neural networks. Data therefore feeds AI and any function, that can be analyzed in data, can then be efficiently performed by AI bots. These systems analyze large volumes of data and interconnect complex processes even in cases where the correlation is not obvious. Machine learning is capable of recognizing patterns in very large samples, building a model that incorporates those patterns, and then allowing the model to "learn". As this process is repeated thousands of times, the model improves its predictive ability.

In the era of the 4th Industrial Revolution, technology with the Internet, Cloud Computing, Big Data, Robotics, and Artificial Intelligence as protagonists is radically transforming the world. Behind all these achievements are the well-known mathematical algorithms (Kishore (June 26, 2023). So, with a look towards the future, which is very close to us now, it is almost certain that we will live in a new world combined with machines and robots. And the more mathematical our approach, the greater our chances of success in these new conditions. In Picture 1 we can see how the future of CAS is being shaped by AI-augmented.

The benefits of AI generally include the following:
• Reduce human error. Accuracy increases and errors are reduced with AI.
• Quicker choices and actions. The main benefits of AI are speed and efficiency.
• Recognizing patterns and forecasting the future. Massive amounts of data, fast processing, and strong algorithms are all combined in AI. Accurately predicting future states through pattern recognition is the aim of the system.
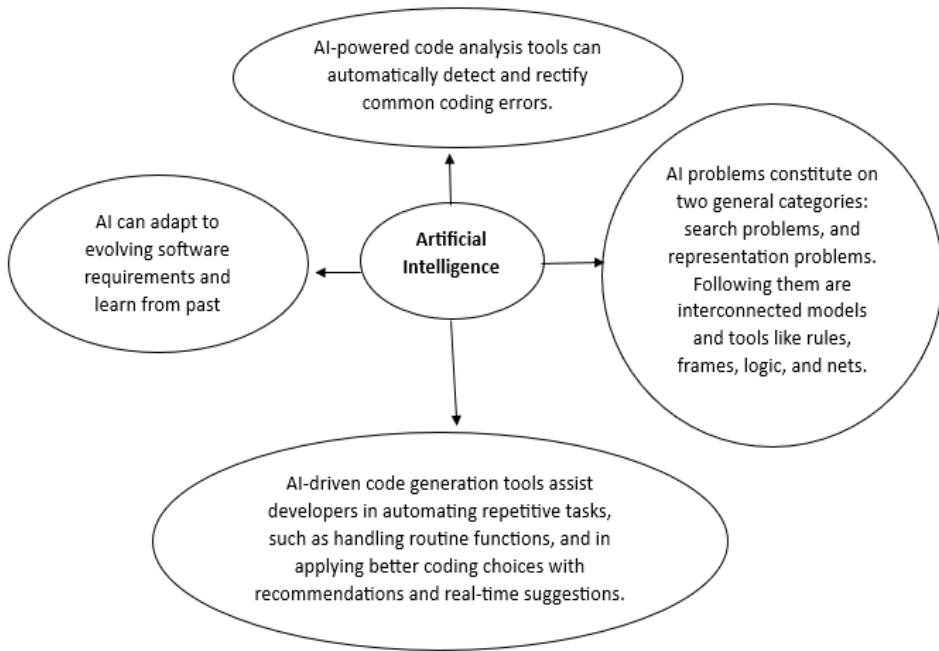
Picture 1. The ways that the future of CAS is being shaped by AI-augmented development
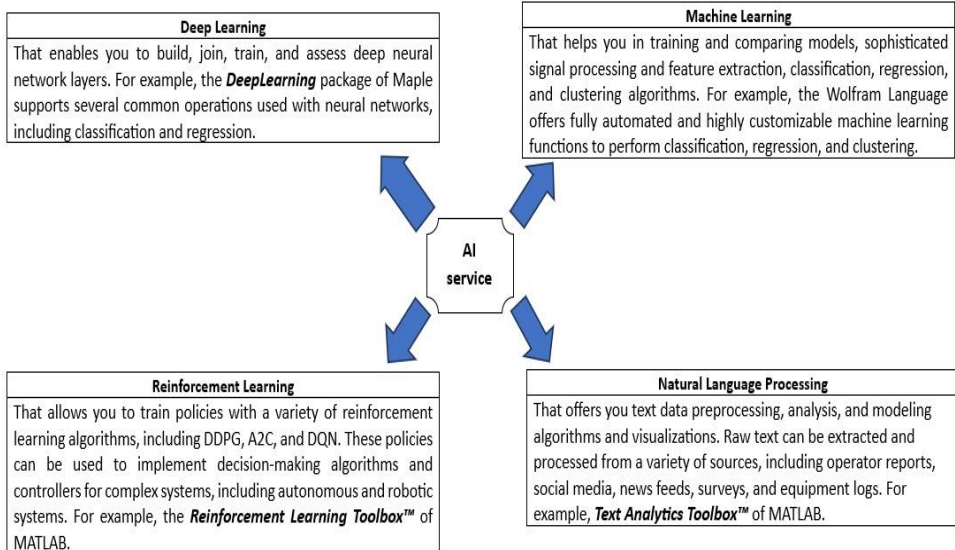
Artificial intelligence and mathematical tools

Mathematical education has changed dramatically with AI-powered tools. Students can learn mathematical concepts in a very different way (without the help of a teacher) studying step-by-step solutions to any kind of mathematical problem. In the following Table 1, we can see some of the most famous Mathematical Tools, Apps, and Platforms.

Table 1. Famous mathematical tools/apps/platforms with AI

Name	Description	Math topics	Cost
Socratic	The user takes a photo of a mathematical problem, and the **App** with AI help, analyzes the problem and gives a step-by-step solution in real time.	Algebra Geometry Calculus Trigonometry Statistics	Free
Microsoft Math Solver *(powered by Microsoft)*	A very famous **intuitive AI-based tool** that gives the user a step-by-step solution for a mathematical problem.	Algebra Geometry Calculus Trigonometry • Statistics	Free
Mathway	It's an online mathematical problem-solving **platform** with AI. It analyzes a mathematical problem and gives a step-by-step solution in real-time.	Algebra Geometry Calculus Trigonometry • Statistics	Only the basic edition is free.
Wolfram Alpha *(from the makers of Mathematica)*	It's a very famous **computational knowledge engine** for various kinds of mathematical problems.	Algebra Geometry Calculus Trigonometry • Statistics	Only the basic edition is free.
Maple Calculator *(from the makers of Maple)*	It's an **online calculator tool** with AI for complex mathematical problems.	Algebra Geometry Calculus Trigonometry • Statistics	Free

Name	Description	Math topics	Cost
Photomath	It's an **App** for students with AI technology. By scanning a math problem, the user gets from the App a step-by-step solution in real time.	Algebra Geometry Calculus Trigonometry • Statistics	Only the basic edition is free.
Symbolab	It's a mathematical **educational tool** with AI for complex mathematical problems.	Algebra Geometry Calculus Trigonometry • Statistics	Only the basic edition is free.
GeoGebra	It's an online mathematical problem-solving **platform** with AI. It specializes in Geometry problems.	Geometry Algebra Calculus	Free
Brilliant	It's a mathematical educational **online platform** with AI technology.	Algebra Geometry Calculus Trigonometry • Statistics	Only the basic edition is free.
Desmos	It's an **online calculator tool** with AI for complex mathematical problems.	Algebra Geometry Calculus Trigonometry • Statistics	Free
SymbMath	It's a mathematical educational tool with AI that performs symbolic mathematics and advanced mathematical calculations.	Algebra Calculus	Free
MyScript	It's **a mathematical App** that uses machine learning to interpret math writings and gives us a step-by-step solution in real time.	Algebra Geometry Calculus Trigonometry • Statistics	Free

In most CASs, some specialized tools and techniques offer us artificial intelligence services. In the following Picture 2, we can see what kind of AI services they offer.

Deep Learning

That enables you to build, join, train, and assess deep neural network layers. For example, the **DeepLearning** package of Maple supports several common operations used with neural networks, including classification and regression.

Machine Learning

That helps you in training and comparing models, sophisticated signal processing and feature extraction, classification, regression, and clustering algorithms. For example, the Wolfram Language offers fully automated and highly customizable machine learning functions to perform classification, regression, and clustering.

AI service

Reinforcement Learning

That allows you to train policies with a variety of reinforcement learning algorithms, including DDPG, A2C, and DQN. These policies can be used to implement decision-making algorithms and controllers for complex systems, including autonomous and robotic systems. For example, the **Reinforcement Learning Toolbox™** of MATLAB.

Natural Language Processing

That offers you text data preprocessing, analysis, and modeling algorithms and visualizations. Raw text can be extracted and processed from a variety of sources, including operator reports, social media, news feeds, surveys, and equipment logs. For example, **Text Analytics Toolbox™** of MATLAB.

Picture 2. Artificial Intelligence Services

The previous tools can provide step-by-step solutions to a great variety of mathematical problems. They are restricted to the things they were taught specifically, though. ChatGPT (November 30, 2022) takes a more statistical approach(probabilistic) and tries to recognize patterns and then make predictions(sometimes it makes incorrect predictions). The language model Chat Generative Pre-Trained Transformer (ChatGPT) is the most famous question-and-answer dialogue system. This technology also represents a significant advancement in the field of code generation. The tool helps developers to automate different parts of software development, saving time and increasing productivity. From March 2023 GPT-4 succeeded ChatGPT. The first one is good for searching for mathematical terms, methods, algorithms, etc., while the second one can also be used as a tool for easy and simple mathematical problems (Frieder et al., 2023).

Increasing the performance of an AI system

Artificial intelligence (AI) derives its "intelligence" from pattern-finding analysis of a given dataset. It is unaware of the world outside of this dataset. It is a multifaceted tool that is revolutionizing various fields by allowing people to reconsider how we combine data, evaluate information, and apply the resulting insights to make better decisions. In order to increase the performance of an artificial intelligence system we can follow the tips below:

a. Provide more information

One of the most popular and efficient ways to increase the accuracy of your machine-learning model is to add new and fresh data. The training of models is one of the main obstacles in the development of AI systems. Training AI and machine learning algorithms require gathering high-quality data and preparing them. This makes it the most crucial phase of the process. Many studies have demonstrated that the accuracy of AI models and dataset size have a positive correlation. Moreover, the AI models need complex datasets to function and are vulnerable to dataset bias (Brenton, 2021; Anyverse, 2022; Liu et al., 2021). Consequently, increasing the dataset that the model is retrained on can be a useful strategy for enhancing AI models.

An essential component of optimizing AI models is calculus. Calculus is used in methods like gradient descent and backpropagation to reduce errors and modify the parameters of machine learning models. AI systems can learn from data and keep getting better at what they do thanks to these mathematical techniques.

b. Enhance the information

Enhancing the current data can also lead to a better AI model. It can be carried out by enriching the data, enhancing the quality of the data, and making use of augmented data.

c. Integrate with current systems in use

Ascertain how AI systems will work with the tools and processes you already use. It's very important to do this because there will exist settings that enhance performance and compatibility between systems.

d. Enhance the algorithm

There are situations when the model's original algorithm needs to be enhanced. This may occur for a variety of reasons, such as a shift in the population the model is applied. So, improving the algorithm by giving it new parameters (data layouts that are cache-friendly and effective algorithms) may be a useful strategy for raising model performance.

Conclusions

The creation and effectiveness of AI systems depend heavily on the use of CASs. The concepts and tools needed for AI algorithms to process data, identify trends, and reach well-informed conclusions are found in Mathematics. The symbiotic relationship between Mathematics and AI will be essential in opening up new avenues and prospects for innovation as AI develops and changes our world. AI should supplement human abilities, not replace them, so you must take into account how decision-making processes

will evaluate and apply AI outputs. So, to increase the performance of an Artificial Intelligence system it's advisable to enhance the algorithm and information, provide more useful data, and integrate with current systems in use.

New theories, algorithms, and methodologies have been developed as a result of the advancement of technology, particularly Computer science, which has created new opportunities for mathematical exploration. New programming tools and concepts in Mathematics have already been developed as a result of developments in Artificial Intelligence and quantum computing. At a rate never seen before, artificial intelligence is transforming industries and our lives, and Mathematics is essential to this development.

There are dozens of other new techniques being used that may change the way we understand and use Mathematics today. The future of Mathematics is uncertain because all new technologies are running at a very fast pace and are dragging mathematical science along new paths. As long as there are unsolved problems such as the Riemann Hypothesis there will always be the incentive to find new ways to solve them! AI has the potential to revolutionize industries, resolve challenging issues, and improve our daily lives in amazing ways by utilizing the power of CASs.

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